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RESEARCH ARTICLE

Gain enhancement of circular waveguide antennas using near-zero index metamaterials

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Abstract

In this article, a rigorous analytical methodology is introduced for designing near-zero refractive index metamaterials (NZIMs). Our proposed NZIM media is realized by three stacked layers of perforated metallic surfaces, each layer composed of a fishnet-like periodic array of square holes. By a proper design of such structures, a low refractive index medium is achieved at their corresponding plasma frequency. The low refractive index property is studied by retrieving the effective parameters of NZIM via inversion techniques, which gives an effective near-zero refractive index, at an operating frequency of 1.5 GHz. Then, the designed NZIM is used for gain enhancement of a circular waveguide antenna. The analysis shows that the proposed platform can enhance the directivity of our antenna by 3 dB while maintaining the return loss <−20 dB.

KEYWORDS

directivity, inversion algorithm, metamaterial, near-zero refractive index

1 | INTRODUCTION

Artificially constructed metamaterials (MTMs) are capable of manipulating electromagnetic waves, as is not feasible via conventional materials.[1–7] Typically, MTMs are composed of subwavelength inclusions in periodic arrays that interact with electromagnetic waves as a homogenous medium.[8,9] Since the advent of MTMs, these structures have been extensively studied in the framework of microwave applications.[10–12] Among those, many works have been devoted to studying the implication of MTMs in antenna applications.[13–15] Now it is well known that functional characteristics of antennas including gain, directivity, and side-lobe level can be well tuned and improved by MTM's.[16–18] In this context, one of the pioneering works has been done by Enoch et al., which has studied the effect of MTM's on antenna emission.[19] This work has exploited a multilayered wire-grid structure, each layer composed of a fishnet-like periodic array of rectangular holes, to provide a zero refractive index medium. According to the geometric interpretation of Snell's low, such a medium can direct waves nearly perpendicular to the material surface and thus confine the power flow of radiation source to a small solid angle. It is worth mentioning that their work is originally rooted in Pendry's paper,[20] which already had shown that periodic metallic wire arrays, with spacing much smaller than the wavelength, could be effectively described as a homogenous plasma medium in microwave frequencies. These diluted metal structures (wire-grid and wire-media) have been studied both analytically and experimentally, proving the fact that they can be described as a plasma medium. Therefore, similar to natural plasma, which gives zero permittivity in its plasma frequency, diluted metals can also be tailored for achieving a near-zero index metamaterial (NZIM) at a specific frequency.

In this article, we have exploited wire-grid meta-structures for providing an artificial plasma medium of plasma frequency of 1.5 GHz. Then, this artificial plasma is used for enhancing the directivity of radiation in a circular waveguide antenna operating at a narrow bandwidth around 1.5 GHz. Simulations are performed throughout the study by commercial software CST.

2 | NZIM STRUCTURE

Our proposed meta-structure for achieving NZIM is shown in Figure 1. The structure is composed of three layers of metallic wire-grids, each one separated by h from its adjacent layer. Wire-grid layers are composed of 6×6 periodic

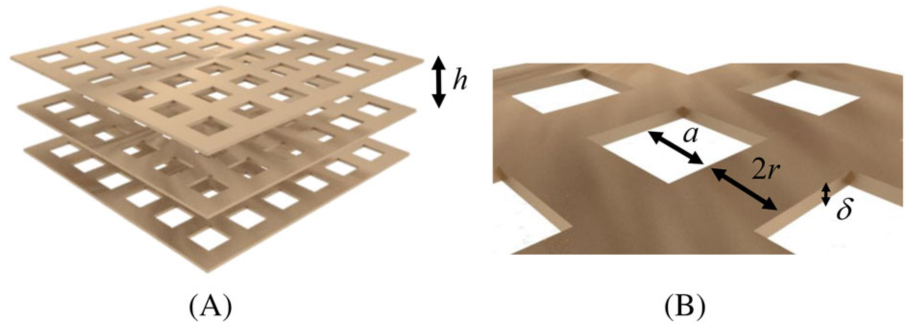

FIGURE 1 A, Three layers of metallic wire-grids for achieving NZIM. h denotes the spacing between layers. B, Unit cell dimensions for each layer. a, r, and δ denote the side length of square holes, the width of stripes, and strip's thickness, respectively [Color figure can be viewed at wileyonlinelibrary.com]

arrays of square holes. The side length of square holes and the width of wires stripes are a and $2r$, respectivelyMTM as shown in Figure 1B, which makes each layer, a periodic grid of lattice constant $(a + 2r)$ in x and y directions. Assuming that the spacing between square holes and between adjacent layers is much shorter than the wavelength, the structure can be described as a homogenous matter. In this case, the effective permittivity of structure can be expressed as[20]

$$\varepsilon_{\mathrm{e}} = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}, \tag{1}$$

where ω and ω_{p} are the angular frequency and equivalent plasma frequency of the structure, respectively. The equivalent plasma frequency of so-attained meta-structure can be obtained approximately by the following analytical formula:

$$\omega_{\mathrm{p}}^{-2} = \frac{k(a+r)\,h\,\ln[h(1+a/r)/\delta]}{\pi c_0^2}, \tag{2}$$

where c_0 is the light speed in free space, δ is the thickness of metallic stripes, and k is the revised coefficient which takes a value between 3 and 5. According to the Snell's refraction law, it is well evident that putting an NZIM slab in front of a radiation source can enhance the directivity of emission. If we aim to enhance the directivity of an antenna by wire-grid platform illustrated in Figure 1, then, the structure must be designed, so that it acquires the same plasma frequency as the operating frequency of the antenna.

This demands a proper value of a, r, h, and δ parameters in Equation 2. Usually, in a simple design, there are not many degrees of freedom over the selection of thickness and radius of wires. Therefore, we choose predefined values of 0.23 and 4.6 mm for δ and r, respectively. This reduces the parameter space in Equation 2 to the selection of h (separation between layers) and a (side length of square holes). Figure 2 illustrates the plasma frequency (ω_{p}) of proposed wire-grid structure in Figure 1, along with some constant contours, which has been calculated according to Equation 2 for different values of a and h. It should be noted that here we considered the revised coefficient of $k = 4$. Noting that our target plasma frequency is 1.5 GHz, any pair values of a and h on 1.5 GHz contour (labeled in Figure 2) can provide an NZIM at this frequency. For example, if we aim to have an NZIM structure at a frequency of 1.5 GHz and grid spacing of $h = 60$ mm, then the side length of the square holes should be $a = 59.1$ mm.

3 | SIMULATION RESULTS

In the previous section, we calculated initial values in the wire-grid structure for achieving a plasma frequency of 1.5 GHz. It is worth mentioning that the analytical expression in Equation 2 gives only an approximation of plasma frequency. More accurate characterization of structure requires retrieval techniques, offering effective electromagnetic parameters of the medium, from full-wave reflection and transmission coefficients.[21,22] In this section, to validate previously analytically calculated values, we retrieve the effective refractive index of structure in Figure 1, with parameters values of $h = 60$ mm and $a = 59$ mm. Figure 3 shows the scattering

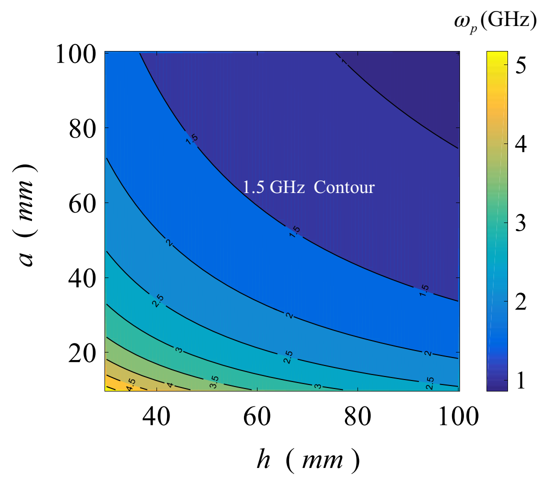

FIGURE 2 Plasma frequency of wire-grid structure along with some constant contours plotted against a (side length of square holes) and h (z spacing between layers) which provides a graphical solution of Equation 2 to obtain design parameters of NZIM structure in Figure 1 [Color figure can be viewed at wileyonlinelibrary.com]

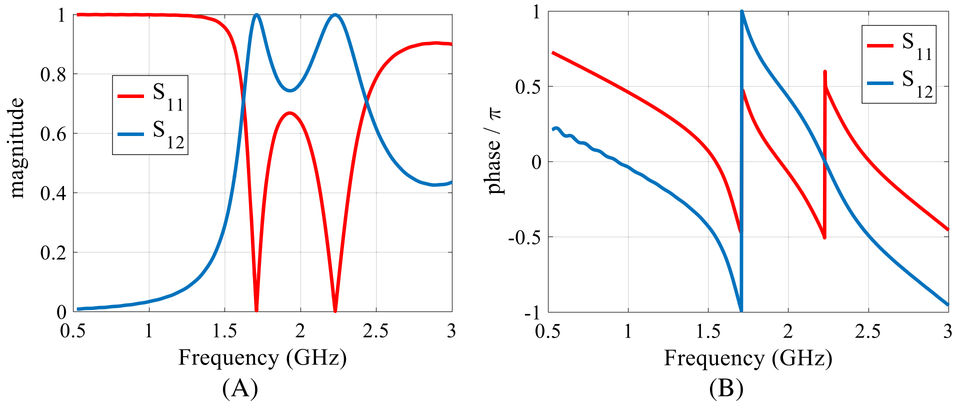

FIGURE 3 (A) Magnitude and (B) phase of scattering parameters of wire-grid media shown in Figure 1, with geometrical dimensions of $a = 59.1$ mm, $h = 60$ mm, $r = 4.6$ mm, and $t = 0.23$ mm [Color figure can be viewed at wileyonlinelibrary.com]

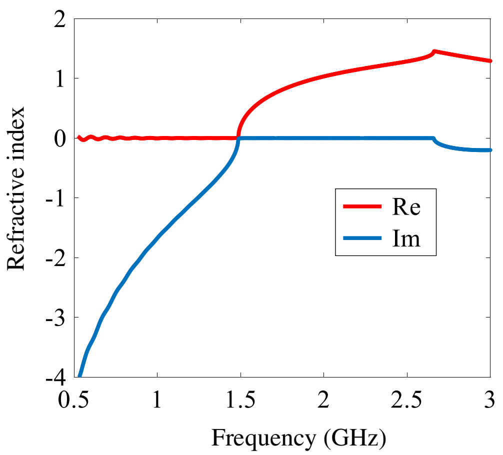

FIGURE 4 Retrieved effective refractive index by inversion algorithm,[20] for wire-grid media shown in Figure 1 with geometrical dimensions of $a = 59.1$ mm, $h = 60$ mm, $r = 4.6$ mm, and $\delta = 0.23$ mm [Color figure can be viewed at wileyonlinelibrary.com]

parameters of structure which are calculated by full-wave numerical simulations.

The effective constitutive parameters can be attributed to the meta-structure, using the aforementioned scattering parameters. For retrieving effective parameters, we exploit Chen's method,[22] which is a very robust and efficient inversion technique. The retrieved refractive index is shown in Figure 4. It

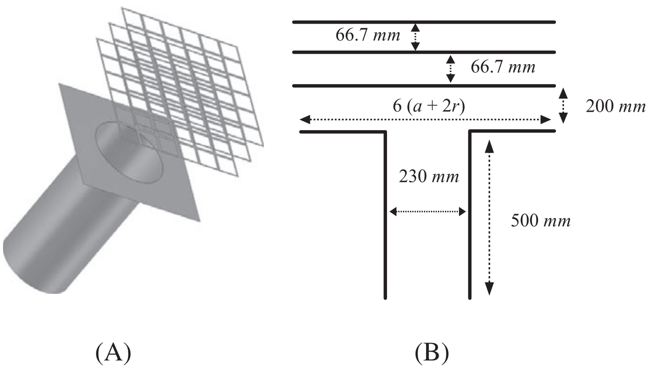

FIGURE 6 A, Final design of wire-grid ZIM structure placed 200 mm above the circular waveguide antenna for achieving directive emission. B, Geometrical dimensions of the structure

can be seen that the plasma frequency of structure ($\simeq$1.4 GHz) is slightly different from the predicted value in Equation 2.

Figure 5 shows the electric field distribution, inside the wire-grid structure for different frequencies. It can be seen that in 0.5 GHz, owing to the large imaginary part of the refractive index, the electromagnetic fields are not able to propagate inside the structure. In 1 GHz, we have a lower value for the imaginary part of permittivity, and thus fields are more progressive in the MTM than 0.5 GHz. In 1.5 GHz, the structure shows its near-zero refractive index

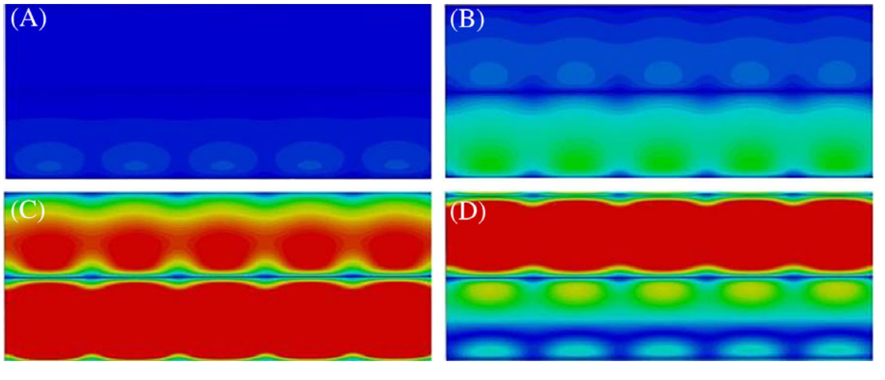

FIGURE 5 The electric field distribution in designed wire-grid ZIM with refractive index shown in Figure 4, at four sample frequencies of (A) 0.5 GHz, (B) 1 GHz, (C) 1.5 GHz, and (D) 1.75 GHz [Color figure can be viewed at wileyonlinelibrary.com]

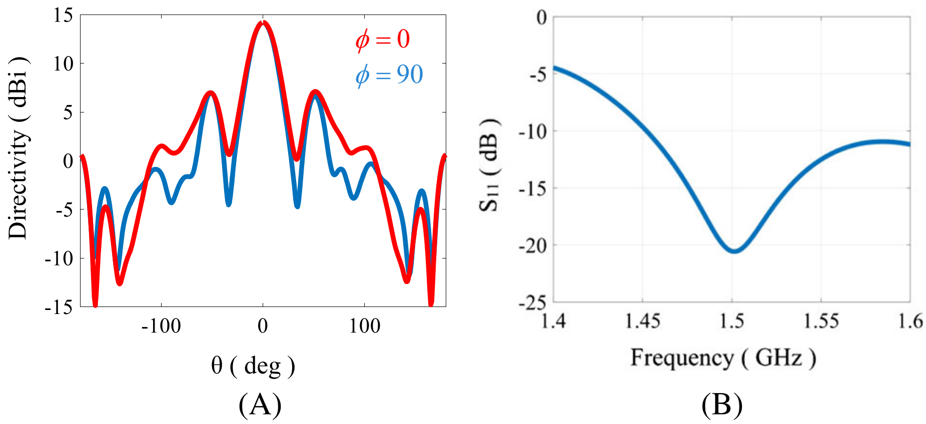

FIGURE 7 A, Directivity of the final design in Figure 6 for constant φ plans. B, S_{11} parameter of the final design in Figure 6 [Color figure can be viewed at wileyonlinelibrary.com]

property which causes nearly constant field distribution inside the material.[23] In 1.75 GHz, wire-grid medium has a small imaginary value and a real-valued refractive index which allows the field to propagate through the structure. All these field configurations are consistent with the previously obtained refractive index in Figure 4.

After this initial design, the optimization methods can be used to move the plasma frequency closer to 1.5 GHz. As mentioned above, we aim to enhance the directivity of a circular waveguide antenna operating at 1.5 GHz, by a zero refractive index MTM. This target antenna is shown in Figure 6A along with NZIM structure which has been placed in front of it.

The antenna is a flanged circular waveguide of length and radius 500 and 115 mm, respectively, which has the directivity of 11.1 dBi when radiating into the free space. Here the ZIM slab is placed 200 mm above the antenna aperture. It should be noted that inserting the NZIM slab very close to the antenna aperture will enhance the return loss and decreases the efficiency of the antenna. In Figure 6B, the final dimensions are shown.

We have assumed that the side length of square holes remains constant as $a = 59.1$ mm and then the optimization process is performed to find the optimum value for h. It can be seen that the optimum value for shifting the plasma resonance to 1.5 GHz has been founded as 66.7 mm.

The radiation pattern of the final design in $\varphi = 0$ and $\varphi = 90$ is shown in Figure 7A.

It is evident that the resulted directivity is 14.1 dBi which is 3 dB higher than the original case. This directivity enhancement is realized by NZIM structure. Finally, we need to check the return loss of the antenna to ensure that is sufficiently low. In Figure 7B, the S_{11} parameter of the antenna is shown which represents very low reflection loss at the frequency of 1.5 GHz.

4 | CONCLUSIONS

In this article, a robust analytical methodology is introduced for designing near-zero refractive index meta-structures (NZIM). For realizing NZIMs, we used perforated metallic planar structures which are stacked on top of each other. The homogenous electromagnetic response of so-attained structures mimics a plasma medium, and therefore can be exploited for achieving zero refractive index at their corresponding plasma frequency. The plasma frequency of such mediums is tuned for operating frequency of the target antenna through analytical and then numerical methods. Then we placed the obtained NZIM slab in front of our target antenna which resulted in 3 dB directivity enhancement while maintaining the return loss <-20 dB.

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