Can light be stopped in realistic metamaterials?

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Tsakmakidis *et al.*[1] extend earlier work (for example, refs 2, 3) in proposing a novel metamaterial waveguide structure that can stop broadband light, producing so-called "trapped rainbows". The authors make the bold assumption that metamaterial loss can be ignored: but material loss, with dispersion, is an inherent feature of negative-index metamaterials (for example, refs 4, 5); any realistic model must include loss and dispersion to satisfy the fundamental principle of causality[6]. Here we revisit the authors' predictions[1] and show that even when an arbitrarily small metamaterial loss is introduced, it is impossible to stop the light; moreover, we find that all slow-light modes in such structures give impractically large propagation losses.

We investigate the TM$_2$ (where TM is transverse-magnetic) mode, analysed by Tsakmakidis *et al.*[1], and calculate the complex waveguide dispersion curves in the presence of metamaterial loss and dispersion. This mode arises from the complex solution to the transcendental equation derived from Maxwell's equations and guidance conditions[7]. Figure 1a shows the computed TM$_2$ dispersion using three different models: (1) neglecting dispersion and loss (dot-dashed grey lines); (2) including dispersion but not loss (dashed grey lines); and (3) including both dispersion and loss (solid black lines). Using the unrealistic (idealized) models (1) or (2), we naturally recover the key result of ref. 1 and obtain a point of zero group velocity at a critical frequency, ω_c, where the group velocity is given by $v_g \equiv d\omega/d\beta$ (here ω is the frequency and β is the propagation constant). In model (2), in addition to the normal mode, there is also a complex mode that was neglected by Tsakmakidis *et al.*[1]: it is bound to the slab but has zero total power flux and a complex propagation constant, $k \equiv \beta + i\alpha$. In model (3), we include the metamaterial loss, which is characterized through the normalized loss parameter, $\tilde{\Gamma} \equiv \Gamma/\omega_c = 0.01$ (see Methods). For this realistic case, the mode dispersion near ω_c changes dramatically: the complex mode splits in two and the normal mode pulls away from its lossless counterpart—rather than having a normal mode and a complex mode, one obtains complex forward-and-backward propagating modes, TM$_2^F$ and TM$_2^B$, respectively. Evidently, the influence of material loss on mode structure is highly non-perturbative near ω_c, where even a tiny amount of loss produces large changes in the dispersion, such that the group velocity is never zero.

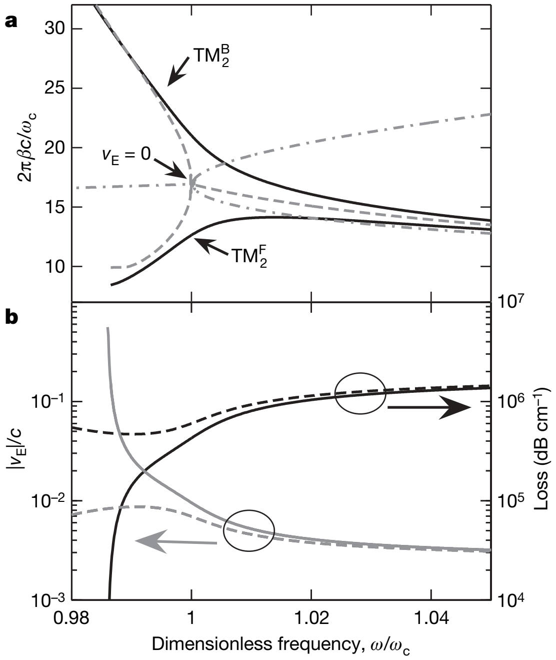

Figure 1 | Dispersion, energy velocity and propagation loss. a, Dispersion curves for the TM$_2$ modes of the waveguide structure of ref. 1. We plot the dispersion for three different models: the lossless and dispersionless model of ref. 1 (dot-dashed grey line), and models including metamaterial dispersion with $\tilde{\Gamma} = 0$ (dashed grey line) and $\tilde{\Gamma} \equiv \Gamma/\omega_c = 0.01$ (solid black line). A loss parameter of $\tilde{\Gamma} = 0.01$ yields a figure-of-merit roughly 30 times better than has been experimentally achieved at optical frequencies and comparable to the state-of-the-art at GHz frequencies[4,5]. **b**, The energy velocity (defined by the ratio of the spatial average of the time-averaged Poynting vector to the spatial average of the time-averaged energy density[12]) and propagation loss versus frequency for the loss model with $\tilde{\Gamma} = 0.01$ for TM$_2^F$ (solid line) and TM$_2^B$ (dashed) modes. Clearly, the energy velocity is never zero and the propagation losses are extremely high whenever the energy velocity is below 0.1c. To better connect to the highlighted optical applications in ref. 1, the propagation loss is calculated for a critical frequency in the optical ($\omega_c/2\pi = 385$ THz). However, our general findings and conclusions scale to GHz and THz. For example, at a frequency of 1 THz, the loss is approximately 1,000 dB cm^{-1} or greater; this is completely impractical as a delay element, given the corresponding energy velocity.

Group velocity and energy flux have been investigated for light propagation in lossy metamaterial waveguides[3,8]; however, as the dispersion relation is complex in lossy systems, the meaning of the group velocity is unclear and the energy velocity (v_E), computed from modes with complex k and real ω, is a better measure of the speed of transport than either v_g or energy flux. We find that, even for arbitrarily small metamaterial loss, the energy velocity is not zero for either mode at any frequency. In Fig. 1b, we plot v_E for $\tilde{\Gamma} = 0.01$; although v_E goes as low as 0.003c (where c is the speed of light in vacuum), it is never zero and the imaginary part of the propagation constant, α, is impractically large. Figure 1b shows the propagation loss for a range of optical frequencies ($\omega_c/2\pi = 385$ THz), which demonstrates a propagation loss >300,000 dB cm^{-1} whenever $v_E < 0.01c$. To put this in context, such losses would result in 200 dB of loss for a pulse delay of only 2.6 ps at optical frequencies. Even if metamaterial losses were reduced by over two orders of magnitude from the state-of-the-art, the losses would be unacceptably high and the slowing of the light would remain unimpressive. Similar calculations for the various modes of this proposed structure and similar structures show comparable—or worse—results for all TM and TE modes.

To ensure a positive energy density, any real metamaterial must have dispersion and thus finite loss at some frequencies. Theoretically, metamaterials are never lossless, even in the presence of gain, except, perhaps, at discrete frequencies[6]. To date, no lossless metamaterial has been found, even at a single frequency.

We have shown that it is impossible to achieve zero-velocity or even practical slow-light modes in metamaterial waveguides over any appreciable bandwidth, so it is impossible to produce "trapped rainbows"[1]. Also, as our findings are important even for arbitrarily small amounts of material loss, attempts to stop light in metallic structures will also fail[9]. Massive losses are also evident in slow-light propagation modes in photonic-crystal waveguides when there is even a tiny amount of unavoidable surface roughness[10]. Surface roughness will also result in additional loss in the metamaterial structure of Tsakmakidis *et al.*[1] and in recently proposed hybrid metamaterial–photonic-crystal structures[11]. In all these cases, the enormous propagation loss and non-zero energy velocity is a direct consequence of the long time that the slow light ultimately spends near the potential absorbers and/or scatterers.

Methods

The permittivity and permeability of the three waveguide layers (top to bottom) are $\varepsilon_i(\omega)$ and $\mu_i(\omega)$ respectively, where $i=\{1,2,3\}$. As in ref. 1, the permittivity and permeability of the cladding layers ($i=\{1,3\}$) are lossless, while for the (realistic) lossy metamaterial we use the standard Drude and Lorentz medium models, respectively, with loss parameter Γ. In all calculations, we use $F=0.6$, $\omega_p=\sqrt{6}\omega_c$, $\omega_o=\sqrt{0.9}\omega_c$ and (as in ref. 1) we take the metamaterial slab thickness to be $d/2=0.55c/\omega_c$; these parameters (defined in ref. 12) have been chosen so as to yield identical results to those presented in ref. 1 for $\omega=\omega_c$ when $\Gamma=0$. In our loss model, we take $\tilde{\Gamma}=0.01$ (refs 4, 5).

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Tsakmakidis et al. *reply*

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Reza *et al.*[1] have confirmed our calculations and results on storing light inside metamaterial waveguides[2]. But they claim that losses constitute an "inherent" feature of any "realistic" negative-refraction metamaterial (NR-MM), and that light can never be stopped inside such a material in a practical way. We argue that both of these assertions are incorrect.

We used a lossless NR-MM core layer, in line with 'perfect' lens and invisibility cloak concepts. Reza *et al.* posit that losses are always present in an NR-MM[3], but this does not imply that a zero-loss NR-MM cannot be designed[4,5]. Those conclusions[3] were based on specific assumptions for the variation of the refractive index imaginary part (n_i) with frequency. Assuming a realistic frequency variation for n_i (but different to the one in ref. 3), based on adjacent absorptive and gain resonances and obeying causality and Kramers-Kronig relations[4], leads to just such a design of a lossless metallic NR-MM, giving a refractive index of $n=-1$. One can also deploy a broadband photonic crystal in its negative-refraction regime, with vanishing losses for a chosen frequency window. Moreover, when electromagnetically induced chirality or some other gain mechanism is deployed it leads to a specific zero-loss frequency window (see ref. 5, for example). Hence the suggestion by Reza *et al.* that the conclusions of ref. 3 are general and applicable to all NR-MM designs, as well as their statements that NR-MMs are never lossless and that no lossless metamaterial has ever been found are, at least, a bit misleading.

We also disagree that light cannot be stopped inside realistic NR-MMs. The mechanism we described for trapping light in NR-MMs relies critically on the presence of negative Goos-Hänchen phase shifts[2], which are also realizable using materials with no negative effective refractive index; our use of a negative-index material in storing THz light shows how this general scheme works. It has been recently shown independently that light can be 'trapped rainbow'-stopped inside realistic NR-MMs, either by using 'spoof' surface plasmons in graded metallic structures—practically lossless in the THz regime[6,7]—or simply by deploying (lossless) photonic crystals in their negative-refraction regime[8]. The former may serve as a way to stop light in metamaterials, by analogy with negative-refraction-enabled subwavelength imaging using plasmonic lenses. Such numerical studies unavoidably model—via the inherent staircase approximations in the representation of the material structure—the influence of surface roughness.

Light does not need to assume precisely zero group (or energy) velocity before it can be practically deployed for slow-light applications and/or regarded as "stored light"[9]. It can be 'trapped rainbow'-stopped inside NR-MMs even in the presence of losses[10,11]. The $30\ \mathrm{dB}\ \mu\mathrm{m}^{-1}$ losses that Reza *et al.* calculate[1] are not at all unusual for slow-light waveguides; ways to drastically reduce higher optical losses, such as $72\ \mathrm{dB}\ \mu\mathrm{m}^{-1}$ (ref. 12) or $50\ \mathrm{dB}\ \mu\mathrm{m}^{-1}$ (ref. 10), which are typical for slow-light plasmonic waveguides, have been reported[12,13]. Also, splitting of the mode band diagram and the loss of the mode degeneracy point[2] in lossy waveguides with negative optical parameters is well-known[14] and has been noted for NR-MM waveguides[15].

The 'trapped rainbow' technique is a powerful method for storing light inside engineered waveguides and is the only way to do so. It should lead to the first experimental demonstration of 'true'—that is, not via atomic or acoustic coherences—storage of photons inside solid-state structures, at room temperature.

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